Statistics

Lecture 11

Feb 19-8:47 AM

SG 19

Clear all lists.

Store 1, 3, and 5 in L1.

Use $\boxed{\text{1-Var Stats}}$ with L1 only to find

$\mu = \bar{x} = \boxed{3}$ $\sigma = \sigma_x = 1.633$ $\sigma^2 = \sigma_x^2 = 2.\overline{6} = \boxed{\frac{8}{3}}$

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4: \sigma_x}$ $\boxed{x^2}$ $\boxed{\text{Enter}}$

Take all Samples of Size 2 with replacement from 1, 3, and 5.

1,1	1,3	1,5
3,1	3,3	3,5
5,1	5,3	5,5

9 Samples of Size 2.

Find $\bar{x}$ of each Sample.

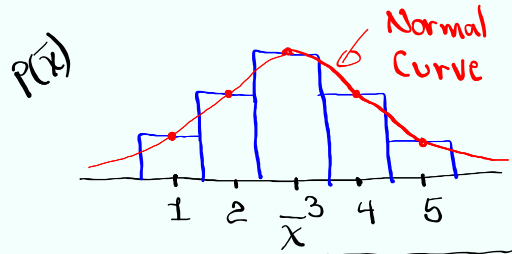
$$\left.\begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{matrix}\right\} \text{9 means}$$

$\bar{x}$	$P(\bar{x})$
1	1/9
2	2/9
3	3/9
4	2/9
5	1/9

Nov 7-11:39 AM

$\bar{x}$	$P(\bar{x})$
1	1/9
2	2/9
3	3/9
4	2/9
5	1/9

Draw Prob. dist. Histogram



$\bar{x} \to L2$, $P(\bar{x}) \to L3$ use $\boxed{\text{1-Var Stats}}$ with L2 & L3. Find

$\mu_{\bar{x}} = \boxed{3}$ $\sigma_{\bar{x}} = 1.155$ $\sigma^2_{\bar{x}} = 1.\bar{3} = \frac{4}{3} = \frac{8}{3} \cdot \frac{1}{2}$

Central limit Theorem (CLT)

$\boxed{\mu_{\bar{x}} = \mu}$ $\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$ $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

Nov 7-11:48 AM

Exam Scores are normally dist. with mean of 82 and Standard dev. of 12.

$N(82, 12)$

If we randomly Select group of $\underbrace{9 \text{ exams}}_{}$. $n=9$

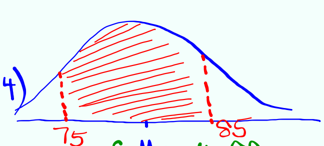
$\underset{\text{CLT}}{\mu_{\bar{x}} = \mu} = \boxed{82}$ $\underset{\text{CLT}}{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}} = \frac{12}{\sqrt{9}} = \frac{12}{3} = \boxed{4}$

Find the Prob. that $\boxed{\text{their mean Score}}$ ($\bar{x}$) is $\underbrace{\text{between 75 and 85}}$.

$P(75 < \bar{x} < 85)$

$= \text{normalcdf}(75, 85, 82, 4)$

$= \boxed{.733} = 73.3\%$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 82 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{9}} = 4 \end{cases}$

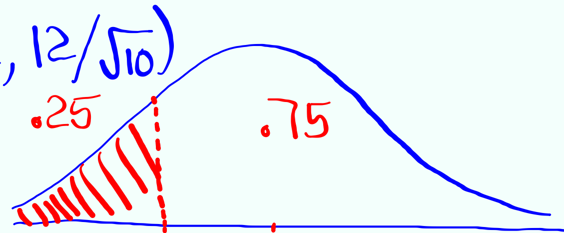
Nov 7-11:56 AM

Find $\bar{x} = Q_1$, rounded to whole #, for randomly selected group of $\underbrace{\text{10 exams}}^{n=10}$.

$\bar{x} = \text{invNorm}(.25, 82, 12/\sqrt{10})$

$= 79.440$

$\approx \boxed{79}$



$\bar{x} = Q_1$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 82 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{12}{\sqrt{10}} \end{cases}$

SG 19 ✓

Nov 7-12:03 PM

SG 20

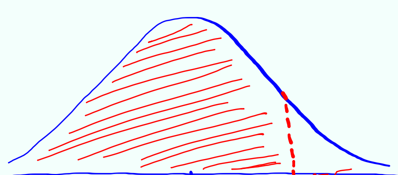
Ages of nurses are normally dist with the mean of 48 yrs and Standard deviation of 8 yrs. $N(48, 8)$

If we randomly Select $\boxed{\text{5 nurses}}^{n=5}$, find the prob. that $\boxed{\text{their mean age}}^{\bar{x}}$ is $\underbrace{\text{below 55 Yrs.}}$

$P(\bar{x} < 55)$

$\boxed{\text{2nd}}\boxed{,}$

$(-)$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 48 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8}{\sqrt{5}} \end{cases}$

$= \text{normalcdf}(-E99, 55, 48, 8/\sqrt{5})$

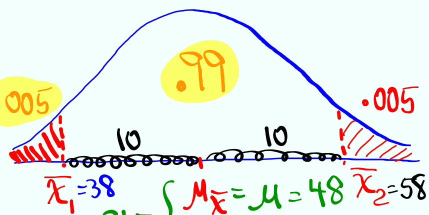
$= \boxed{.975}$

Nov 7-12:08 PM

Find two means, round to whole #, that Separate the $\boxed{\text{middle } 99\%}$ from the rest for randomly selected group of $\boxed{4 \text{ nurses}}$ $n=4$

$1-.99=.01$

$.01 \div 2 = .005$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 48 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8}{\sqrt{4}} = 4 \end{cases}$

$\bar{x}_1 = \text{invNorm}(.005, 48, 4)$

$= 37.697$

$\approx \boxed{38}$

$\bar{x}_2 = \text{invNorm}(.995, 48, 4)$

$= 58.303 \approx \boxed{58}$

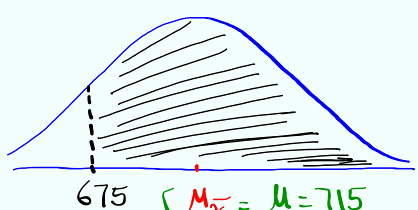
Nov 7-12:15 PM

Credit Scores are normally dist. with mean of 715 and Standard dev. of 75.

$N(715, 75)$

If we randomly Select $\boxed{3 \text{ applications}}$, $n=3$ Find the prop. that $\boxed{\text{their mean credit Score}}$ $\bar{x}$ is $\underbrace{\text{above}}$ 675.

$P(\bar{x} > 675)$



$=\text{normalcdf}(675, E99, 715, 75/\sqrt{3})$

$= \boxed{.822}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 715 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{75}{\sqrt{3}} \end{cases}$

Nov 7-12:23 PM

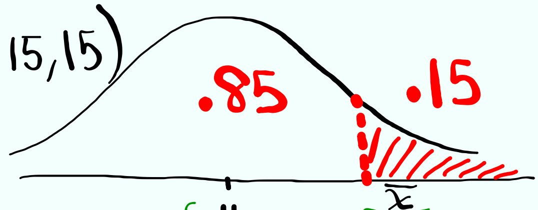
For randomly selected 25 applications, find $\bar{x}$ that separates the top 15% from the rest.

$n=25$

$\bar{x} = P_{85} = \text{invNorm}(.85, 715, 15)$

$= 730.547$

$\approx \boxed{731}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 715 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{75}{\sqrt{25}} = 15 \end{cases}$

SG 20 ✓

Nov 7-12:29 PM

$Z_{\alpha/2}$ is the value with the area of $\alpha/2$ to its right.

Critical Value

SG 21 & SG 22

$\alpha \rightarrow$ Alpha

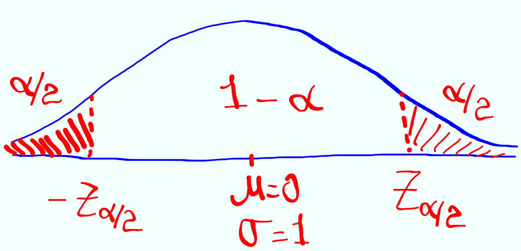
$0 < \alpha < 1$

Significance level

$1-\alpha$ Middle Area

$(1-\alpha)\cdot 100\%$ Confidence level

$\alpha/2$ is the area on the right tail & the left tail.



How to find $Z_{\alpha/2}$

Use invNorm

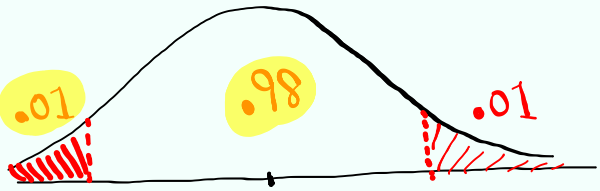
Nov 7-12:48 PM

Find $z_{.01}$ ← Right-Tail Area

$\frac{\alpha}{2} = .01$ $\alpha = .02$ Significance level

$1-\alpha = .98$ Middle Area

$.98(100)\% = 98\%$ Conf. level



$-z_{.01}$ $\mu = 0$ $z_{.01} = 2.326$

-2.326 $\sigma = 1$

$z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

Nov 7-12:53 PM

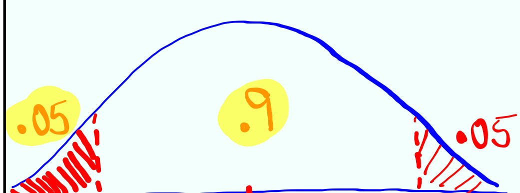
Find $\pm z_{\alpha/2}$ for $\alpha = .1$

Significance level

$\alpha/2 = .05$ area on each tail

$1-\alpha = .9$ Middle Area

$.9(100)\% = 90\%$ Conf. level



$-z_{.05}$ $\mu = 0$ $z_{.05}$

$\sigma = 1$

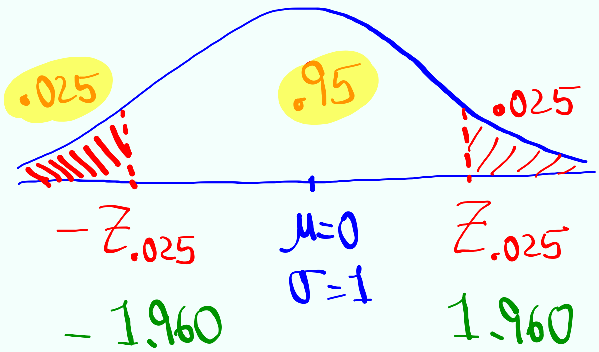
-1.645 1.645

$z_{.05} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

Nov 7-12:58 PM

Find $\pm Z_{\alpha/2}$ for 95% conf. level.

$\underbrace{\text{95\% conf. level}}_{\text{Middle Area } .95}$



$1-\alpha = .95$

$\alpha = .05$ Sig. level

$\alpha/2 = .025$ area of each tail.

$$Z_{.025} = \text{invNorm}(.975, 0, 1) \approx \boxed{1.960}$$

Nov 7-1:03 PM

$t_{\alpha/2}$ t - Dist.

1) Bell-shape, Symmetric, Total area = 1

2) $\mu=0$, σ is unknown

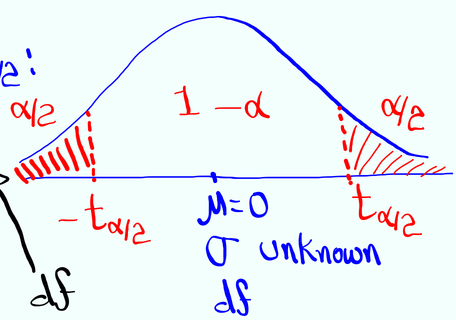
3) It comes with degrees of freedom (df)

How to find $t_{\alpha/2}$:

Use invT(,)

Left Area

df



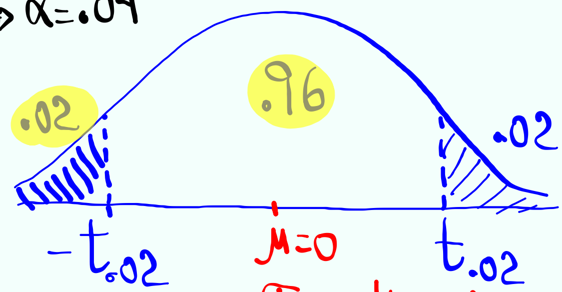
Nov 7-1:08 PM

Find $t_{.02}$ with $df=9$.

$t_{.02}$ ← Right are

$\frac{\alpha}{2} = .02 \rightarrow \alpha = .04$

$1-\alpha = .96$ Middle Area



σ unknown

$df=9$

-2.398 $\quad$ 2.398

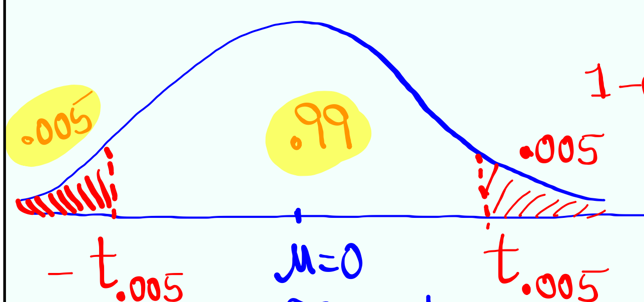
$t_{.02} = \text{invt}(.98, 9) = \boxed{2.398}$

Nov 7-1:12 PM

Find $\pm t_{\alpha/2}$ with $\alpha = .01$ & $df=24$.

$\alpha/2 = .005$ Area of each tail

$1-\alpha = .99$ Middle area



-2.797

σ unknown

$df=24$

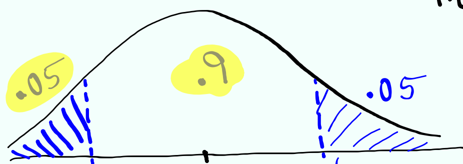
$t_{.005} = \text{invT}(.995, 24)$

$= \boxed{2.797}$

Nov 7-1:17 PM

Find $\pm t_{\alpha/2}$ for 90% $\underbrace{\text{Conf. level}}_{\text{C-level}}$ with df=99.

Middle Area = .9



$1-\alpha=.9$

$\alpha=.1$ Sig. level

$\alpha/2=.05$ area on each tail.

$-t_{.05}$ $\mu=0$ $t_{.05}$

-1.660 σ unknown 1.660

df=99

$t_{.05}=\text{invT}(.95, 99)=\boxed{1.660}$

As df increases,

$t_{\alpha/2}\approx Z_{\alpha/2}$

Nov 7-1:20 PM

What is degrees of freedom?

25 Students, I bring 25 Donut,

First	Student	25 choices
Second	"	24 "
Third	"	23 "
⋮		
Last	Student	(0 choices) 1 donut left

df=24

7 clean Shirts

Monday	7 choices
Tuesday	6 choices
⋮	
Sunday	0 choices (1 clean shirt)

df=6

df will be determined by topics or it will be given to us.

Nov 7-1:26 PM

Estimating Parameters: SG 21

To estimate Population we use Sample

Proportion	p	Point-estimate: Proportion	$\hat{p}$
Mean	μ	Point-estimate: Mean	$\bar{x}$
Standard dev.	σ	Standard dev.	s

to estimate, Answer is range of Value which is called Confidence Interval.

Every estimation comes with Confidence level. If none is given, we use 95%

Nov 7-1:42 PM

Estimating Population Proportion P:

Answer $\quad < p <$

$$\hat{p} - E < p < \hat{p} + E$$

Sample Proportion

Margin of error

$$\hat{p} = \frac{x}{n}$$

x ← # of favorable responces

n ← Sample Size

$$\hat{q} = 1 - \hat{p}$$

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$$

Critical value for $(1-\alpha)\cdot 100\%$ C-level.

Conf. level

Nov 7-1:49 PM

I Surveyed 100 Students, 80 of them had iPhone. $n=100$ $x=80$

$\hat{p}=\frac{x}{n}=\frac{80}{100}=.8$

$\hat{q}=1-\hat{p}=.2$

C-level = 90%

Find $\underbrace{\text{90\% Conf. interval}}$ for the prop. of all students that have iPhone.

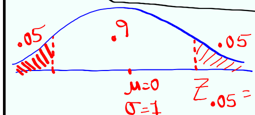
$E=Z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$=1.645\cdot\sqrt{\frac{(.8)(.2)}{100}}\approx\boxed{.07}$

$\hat{p}-E<p<\hat{p}+E$

$.8-.07<p<.8+.07$

$\boxed{.73<p<.87}$



$Z_{.05}=\text{invNorm}(.95,0,1)=\boxed{1.645}$

STAT

TESTS

1-PropZInt

$x=80$

$n=100$

C-level: .9

Calculate

I am 90% Confident that between 73% & 87% of all students have iPhone.

$.73<p<.87$

Nov 7-1:54 PM

I Surveyed 150 Students, 20% of them were Smokers. $n=150$

$\hat{p}=.2$ $\rightarrow x=n\hat{p}=150(.2)=30$

if decimal → Round-up

C-level: .99

Find $\underbrace{\text{99\% Conf. interval}}$ for the prop. of all Students that Smoke.

STAT TESTS

1-PropZInt

$x=30$

$n=150$

C-level: .99

Calculate

$\boxed{.12<p<.28}$

$E=\frac{.28-.12}{2}=\boxed{.08}$

$\hat{p}=\frac{.28+.12}{2}=\boxed{.2}$

Nov 7-2:05 PM

I surveyed 75 students, 6% of them were left handed. $n=75$

$\hat{p}=.06$ $\rightarrow X=n\hat{p}=75(.06)=4.5\approx\boxed{5}$

No C-level $\Rightarrow$ use .95

Find conf. interval for the prop. of all students that are left-handed.

1-PropZInt

$x=5$

$n=75$

C-level: .95

$\boxed{.01<p<.12}$

$$E=\frac{.12-.01}{2}=.055\approx5.5\%$$

$$\hat{p}=\frac{.12+.01}{2}=.065\approx6.5\%$$

Nov 7-2:12 PM

How to determine the sample size:

$$E=Z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

with some algebra, we isolate n

$$n=\hat{p}\,\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

if decimal $\rightarrow$ Round-up

If $\hat{p}$ & $\hat{q}$ are unknown $\rightarrow$ use .5 for each

$$n=.25\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

Nov 7-2:19 PM

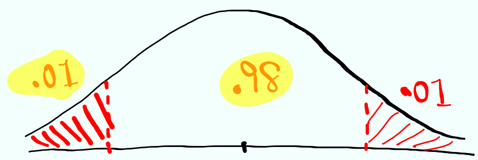
Given $\hat{p} = .4$, $E = .06$, C-level: .98

Find min. Sample Size needed.

$$n = \hat{p}\,\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2 = (.4)(.6)\left(\frac{2.326}{.06}\right)^2$$

$= 360.685$

$\approx \boxed{361}$



$\mu = 0$
$\sigma = 1$

$Z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

Redo with $E = .08$

$$n = (.4)(.6)\left(\frac{2.326}{.08}\right)^2 \approx \boxed{203}$$

Nov 7-2:22 PM

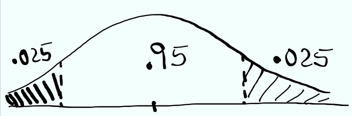
Find min. Sample Size (n) needed to Construct Conf. interval for pop. Prop. and error not exceed 10%.

no C-level → .95

$E = .1$

$\hat{p}$ unknown

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.960}{.1}\right)^2 \approx \boxed{97}$$



$\mu = 0$
$\sigma = 1$

$Z_{.025} = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

Redo with $E = .05$

$$n = .25\left(\frac{1.96}{.05}\right)^2 \approx \boxed{385}$$

Nov 7-2:30 PM